THE SUM OF ALL NATURAL NUMBERS

A while ago, I was surfing the internet and came across a YouTube channel called numberphile. Here, I found a video titled: ASTOUNDING: 1+2+3+4…. = -1/12. I stared at it for a few seconds, contemplating life and the existence of mathematics. Disbelief, a word to describe it. Then I clicked, so here I am going to prove that the sum of all natural numbers is equal to -1/12.

Let's start with an equation:

$$\sum_{n=1}^{\infty} n = -\frac{1}{12}$$

We start with 3 different sequences:

$$S_{1=1-1+1-1+1-1+1\ldots\ldots}$$

$$S_{2=1-2+3-4+5-6+7\ldots\ldots\ldots}$$

$$S = 1 + 2 + 3 + 4 + 5 \ldots \ldots ..$$

S, being our original sequence.

Let's solve these sequences

In the first one, we can clearly see that each odd succession the term is 1 and each even succession is 0 (1 – 1). If this does extend to infinity, which is not determined to be odd or even, then we wouldn't know whether it is a 1 or a 0. So, we take the average of this sequence which is ½.

Hence,

$$S_{1=1-1+1-1+1-1+1\ldots\ldots} = \frac{1}{2}$$

For the second equation, the process is a bit less intuitive. We need to first duplicate the equation and add them together. So we get,

$$\begin{aligned} 2S_2 \quad &= 1-2+3-4+5-6+7\ldots\ldots\ldots \\ &+ \quad 1-2+3-4+5-6+7\ldots\ldots\ldots \end{aligned}$$

This gives us the sequence, 1-1+1-1+1-1+1…. . This is clearly the first sequence, so we can form an equation.

$$2S_{2=\frac{1}{2}}$$

As we know, S_1 = 1/2

$$S_{2=\frac{1}{2}\times\frac{1}{2}}$$

Hence,

$$S_{2=\frac{1}{4}}$$

Finally, we move on to our main sequence and the title for this argument as we have everything we need to prove this crazy idea. We need to subtract sequence
2 from our main sequence $S - S_2$.

So we get, $S - S_2 = 1 + 2 + 3 + 4 + 5 + 6 \ldots \ldots$
- [1 - 2 + 3 - 4 + 5 - 6]……..

And the end result is 0, 4, 0, 8, 0, 12…..

Multiples of 4, so we now factorise this to get

4(1 + 2 + 3…..)

This is our main sequence. We now solve this equation.

$$S - S_{2=}4S$$

$$3S = -\frac{1}{4}$$

And finally,

$$S = -\frac{1}{4} \times \frac{1}{3}$$

$$S = -\frac{1}{12}$$

Although, this idea is quite non-intuitive it is used by many physicist in the idea of string theory and the existence of the universe.

So, there we have proof of the sum of all natural numbers and its astonishing result.